Book of Abstracts

Workshop
Recent Advances in Kinetic Theory and Fluid Dynamics Models

in honour of Claude Bardos

Organizers: Piotr Gwiazda, Jakub Skrzeczkowski,
Agnieszka Świerczewska-Gwiazda, Edriss S. Titi

Stochastic Lagrangian perturbation of Lie transport and applications to disspative fluids

Nicolas Besse

Université Côte d'Azur

Abstract

In this talk we propose a novel stochastic Lagrangian formulation of dissipatively perturbed Lie transport, which is based on the "statistical generalized Cauchy invariant equation". This formulation consists of, first, finding a convenient Lagrangian formulation of the Lie transport equations involving particle trajectories, for instance the backward generalized Cauchy invariants equation, and second, performing a stochastic perturbation of the velocity-driven particle trajectories by adding to them white noises. Finally, using Itô's calculus, an ensemble average of the stochastically perturbated generalized Cauchy invariant equations allows us to obtain the Lie transport equations perturbed by a deterministic term given by the sum of squares of some Lie derivative operators. A remarkable property of these equations is that they satisfy a statistical Kelvin–Helmholtz theorem on conservation of circulation and flux. Moreover these results are obtained in periodic Euclidean spaces as well as on closed Riemannian manifolds. In particular, we recover and thus generalize the Constantin–Iyer results for the incompressible Navier–Stokes equation to a larger class of (deterministic) dissipative PDEs. A first application of this stochastic Lagrangian formulation is the derivation of new Lagrangian formulations for non-ideal hydrodynamic and magnetohydrodynamic models in flat and curved spaces, and in particular we obtain the stochastic-Lagrangian incompressible extended MHD equations. As a second application, we use this new stochastic Lagrangian formulation to study the local well-posedness, the non-resistive limit and the global existence of classical solutions for the non-ideal incompressible extended MHD in the flat torus.

Einstein's equations in vacuum viewed as a matrix-valued generalization of the Euler equations of isothermal gases

Yann Brenier

CNRS and Université Paris-Saclay

Abstract

Einstein's equations in vacuum can be recovered from a variational principle strikingly similar to the one needed to get the isothermal Euler equations with a correspondance between the cosmological constant and the speed of sound. This requires the introduction of a suitable phase space, just as in kinetic theory, to express the Einstein equations as a kind of generalized, matrix-valued and kinetic, version of the isothermal Euler equations.

Pressure, Structure Functions and Singularities

Peter Constantin

Princeton University

Abstract

I will describe sufficient conditions for regularity of solutions of incompressible Euler and Navier-Stokes equations based on the hydrodynamic pressure and on structure functions.

Incompressible limit for a two-species tumour growth model

Tomasz Dębiec

Sorbonne Université

Abstract

We study a two-species advection-reaction system, a well-known model with applications in modelling tumour growth. The cell densities are advected by the gradient of a chemical potential which satisfies the so-called Brinkman law, while the growth rate of each population is governed by a function of the joint population pressure. We present a rigorous argument on connecting the coupled PDE system to a more geometric formulation, wherein the total population density is limited to a critical value and the pressure vanishes on unsaturated regions. Joint work with B. Perthame, M. Schmidtchen and N. Vauchelet.

From Vlasov equation to degenerate nonlocal Cahn-Hilliard equation

Charles Elbar

Sorbonne Université

Abstract

We provide a rigorous mathematical framework to establish the hydrodynamic limit of the Vlasov model introduced by Noguchi and Takata in order to describe phase transition of fluids by kinetic equations. We prove that, when the scale parameter tends to 0, this model converges to a nonlocal Cahn-Hilliard equation with degenerate mobility. For our analysis, we introduce apropriate forms of the short and long range potentials which allow us to derive Helmhotlz free energy estimates. Several compactness properties follow from the energy, the energy dissipation and kinetic averaging lemmas. In particular we prove a new weak compactness bound on the current. Joint work with Marco Mason, Benoît Perthame and Jakub Skrzeczkowski.

Intermittency in turbulence and the 3D Navier-Stokes regularity problem

Aseel Farhat

Florida State University

Abstract

We describe several aspects of an analytic/geometric framework for the three-dimensional Navier-Stokes regularity problem, which is directly inspired by the morphology of the regions of intense vorticity/velocity gradients observed in computational simulations of three-dimensional turbulence. Among these, we present a proof that the hyper-dissipative 3D Navier-Stokes are regular within an appropriate functional setting incorporating the intermittency in turbulent regimes, with any power of the Laplacian greater than 1.

Radiative Transfer in Fluids: From Mathematical Analysis to Numerical Simulations

François Golse

Ecole polytechnique

Abstract

This talk presents a simplified model for radiative transfer in a fluid governed by the Navier-Stokes equations. We study the mathematical properties of this model, together with algorithms to compute its solutions. Examples of numerical simulations based on these algorithms will be presented. Work in collaboration with O. Pironneau and C. Bardos.

On implicit constitutive relations to parabolic problems

Erika Maringová

Institute of Science and Technology Austria

Abstract

This is a joint work with Josef Málek and Miroslav Bulíček (Charles University, Prague, Czech Republic). We study systems of nonlinear partial differential equations of parabolic type, in which the elliptic operator is replaced by the first order divergence operator acting on a flux function, which is related to the spatial gradient of the unknown through an additional implicit equation. Formulating four conditions concerning the form of the implicit equation, we first show that these conditions describe a maximal monotone p-coercive graph. We then establish the global-in-time and large-data existence of a (weak) solution and its uniqueness. The theory is tractable from the point of view of numerical approximations. For details, we refer to:

M. Bulíček, E. Maringová, J. Málek, *On nonlinear problems of parabolic type with implicit constitutive equations involving flux*, Math. Models Methods Appl. Sci. 31 (2021), no.10, 2039–2090.

Reversal in the Stationary Prandtl Equations

Nader Masmoudi

Courant Institute, New York University

Abstract

We review some recent advances in the stability theory of the stationary Prandtl system in the presence of a reversal flow. Joint work with S. Iyer.

Quantum/Generalized HydroDynamics

Norbert Mauser

Wolfgang Pauli Institute, University of Vienna

Abstract

We discuss "Generalized HydroDynamics (GHD)", a model that has recently attracted a lot of interest among quantum physicists e.g. for the description of quasi 1-d ultra cold atoms. Despite the name it is indeed a kinetic phase space model. By means of a "Thermodynamic Bethe Ansatz" coarse grained (relaxation) dynamics are described by a transport type equation where "quasi momenta" called "rapidities" play the role of the kinetic variable. We present the (physics of the) model, numerical methods, applications and extensions and we discuss the "quantum hydrodynamics" for comparison that are better known in the mathematical kinetic community. Ongoing work with F. Møller, I. Mazets, J. Schmiedmayer, H.P. Stimming and N. Besse.

Semiclassical limit for the Pauli-Poisswell: by Wigner and WKB to Vlasov and Euler

Jakob Möller

Universität Wien

Abstract

The self-consistent Pauli-Poisswell equation, introduced by N. Masmoudi & Mauser, for 2-spinors is a semi-relativistic quantum model for the dynamic of fast electrons. It consists of a 2-vector-valued magnetic Schrödinger equation with an extra term coupling spin and magnetic field via the 2×2 Pauli matrices which is coupled to $1 + 3$ Poisson type equations as the magnetostatic approximation of Maxwell's equations. The Pauli-Poiswell equation is a consistent $O(1/c)$ model, with c being the speed of light, that keeps both relativistic effects magnetism and spin which are both absent in the non-relativistic Schrödinger-Poisson equation.

In this talk we sketch results for the (asymptotic) analysis (for vanishing Planck constant): (a) Wigner transform methods for global in time convergence towards Vlasov-Poisswell kinetic equations. We extend the results of Markowich & Mauser and P.-L. Lions & Paul, using a density matrix formulation with an additional assumption on the statistical weights that allows to gain the regularity to control the current and magnetic potential.
(b) WKB methods for short time convergence towards Euler-Poisswell hydrodynamic equations. We extend the results of P. Zhang and Carles & Alazard, using a priori estimates for the (magnetic) energy of the Pauli-Poiswell-WKB equation taking into account the Poisson equations for the magnetic potential.

We report ongoing joint work with N. Mauser and Changhe Yang, as well as P. Gérard and P. Germain.

New construction of weak solutions to the compressible Navier-Stokes equations

Piotr Bogusław Mucha

University of Warsaw

Abstract

I want to present an idea of the novel construction of weak solutions to the compressible Navier-Stokes equation in the barotropic regime. The critical point is to avoid the approximation by $-\Delta\rho$ of the continuity equation. Instead of that standard approach, a mollification of the pressure is applied. It simplifies significantly the first step of the approximation, and due to its form, we can use the Bresch-Jabin technique to pass to the limit. The result is based on the collaboration with Ewelina Zatorska and Nilasis Chaudhuri.

Viscous MHD vorticity-current equations with data in $L^1(\mathbb{R}^2)$

Marco Sammartino

Università degli Studi di Palermo

Abstract

In this talk we shall consider the 2D viscous MHD equations in the vorticity-current formulation. The initial datum will be supposed to be in L^1. We shall prove that the solution exists globally in time, is unique, and smooth for $t > 0$. This is joint work with V. Sciacca and M. Schonbek.

Degenerate Cahn-Hillard equation: from non-local to local

Jakub Skrzeczkowski

Institute of Mathematics of Polish Academy of Sciences/University of Warsaw

Abstract

We prove rigorously the convergence of a nonlocal Cahn-Hilliard equation with degenerate mobility to a local one on the torus. The proof relies on the compactness result of Bourgain-Brezis-Mironescu and Ponce which allows to deduce compactness from certain non-local energies. Finally, we discuss the main difficulties related to the similar result on a bounded domain. Joint work with Charles Elbar (Sorbonne Université).

Weak solutions for the Stokes system for compressible non-Newtonian fluids with unbounded divergence

Maja Szlenk

University of Warsaw

Abstract

We investigate the existence of weak solutions to a certain system of partial differential equations modeling the behavior of a compressible non-Newtonian fluid for small Reynolds number. We construct the weak solutions despite the lack of the L^∞ estimate on the divergence of the velocity field. The result was obtained by combining the regularity theory for singular operators with a certain logarithmic integral inequality for BMO functions, which allowed us to adjust the method from the recent paper of Feireisl et al. (2015) to more relaxed conditions on the velocity.

On a Multiconfiguration Model of Quantum Chemistry

Saber Trabelsi

Texas A&M University at Qatar

Abstract

The multiconfiguration methods are a natural generalization of well-known simple models for approximating the linear N body Schrödinger equation for atomic and molecular systems with binary (Coulomb in realistic situations) interactions, like the Hartree and the Hartree-Fock methods. These models are intensively used by physicists and chemists for numerical simulations in quantum chemistry for instance. In this talk the formal derivation of the equations and their formulation in a convenient way for the mathematical analysis will be sketched, and several rigorous results will be presented.

Boundary vorticity estimate for the Navier-Stokes equation and control of layer separation in the inviscid limit

Alexis Vasseur

The University of Texas at Austin

Abstract

We provide a new boundary estimate on the vorticity for the incompressible Navier-Stokes equation endowed with no-slip boundary condition. The estimate is rescalable through the inviscid limit. It provides a control on the layer separation at the inviscid Kato double limit. This control is consistent with the layer separation predictions via convex integration.

Some Remarks on Vanishing Viscosity, Anomalous Dissipation, and Boundary Effects

Emil Wiedemann

University of Ulm

Abstract

I give an overview of some results, obtained during the past ten years jointly with Claude Bardos and other collaborators, on various related problems in the mathematical analysis of the Euler equations. This includes the vanishing viscosity limit as a possible selection criterion for non-smooth solutions, weak-strong uniqueness, the conservation or non-conservation of energy for weak solutions, and the effect of physical boundaries in the analysis of these problems.

Weak-strong uniqueness for density dependent, incompressible, non-Newtonian fluids

Jakub Woźnicki

University of Warsaw

Abstract

We analyze the system of the form

$$\begin{aligned} \partial_t \rho + \mathrm{div}_x(\rho u) = 0 \\ \partial_t(\rho u) + \mathrm{div}_x(\rho u \otimes u) + \nabla_x p = \mathrm{div}_x \mathbb{S} \\ \mathrm{div}_x(u) = 0 \end{aligned}$$

where ρ is the mass density, u denotes velocity field, $\mathbb{S}$ the stress tensor and p is the pressure. We are interested in the weak solutions to those equations and prove the weak-strong uniqueness property. This work bases its assumptions on the recent paper by Abbatiello and Feireisl [1], but differs from it in density dependency. Surprisingly, the solutions are not defined by the Young measures, but by the similar tool to the so-called defect measure; we will talk about the differences in the approach.

References

[1] A. Abbatiello and E. Feireisl. *On a class of generalized solutions to equations describing incompressible viscous fluids.* Ann. Mat. Pura Appl. (4), 199(3):1183–1195, 2020.

From compressible Navier–Stokes with nonlocal forces to Euler - relative entropy method

Aneta Wróblewska-Kamińska

Institute of Mathematics of Polish Academy of Sciences

Abstract

We show that weak solutions of degenerate Navier–Stokes equations converge to the strong solutions of the pressureless Euler system with linear drag term, Newtonian repulsion and quadratic confinement. The proof is based on the relative entropy method using the artificial velocity formulation for the one-dimensional Navier–Stokes system. The result is based on the joint work with José A. Carrillo and Ewelina Zatorska.

Moreover we will shortly discuss how to obtain general nonlinear aggregation-diffusion models, including Keller–Segel type models with nonlinear diffusions, as relaxations from nonlocal compressible Euler type hydrodynamic systems via the relative entropy method. This result is based on the joint work with José A. Carrillo and Yinping Peng.

References

[1] J.A. Carrillo, A. Wróblewska-Kamińska, E. Zatorska. Pressureless Euler with nonlocal interactions as a singular limit of degenerate Navier-Stokes system. *J. Math. Anal. Appl.* 492 (2020), no. 1, 124400, 27 pp.

[2] J.A. Carrillo, Y. Peng, A. Wróblewska-Kamińska. Relative Entropy Method for the relaxation limit of Hydrodynamic models. *Mathematical Models for Collective Dynamics of Networks and Heterogeneous Media*, 2020.